Quantum physics. Department of physics. 7th semester.

Lesson №11. ***Movement in central field: Spherical waves, particle in spherically symmetrical finite depth rectangular well***

1. Hometask check.

Task 1. Find expectation value $\left\langle r^{-1}\right\rangle$ in state, which is described by wave function $\psi(r,\theta,\varphi)=Ae^{-r}$. Find $\left\langle \hat{\vec{L}}^2\right\rangle$ and $\left\langle \hat{L}_z\right\rangle$ for this state.

2. Hamiltonian of the particle, which moves in the central field

$$\hat{H}=-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\Delta_{\theta\varphi}\right]+U(r)=-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{\hbar^2\hat{\vec{l}}^2}{2\mu r^2}+U(r);$$

$$\Delta_{\theta\varphi}=\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]=-\hat{\vec{l}}^2.$$

Task 2. Investigate stationary states of a free particle with definite values of energy, angular momentum l and it's z-projection m (spherical waves). (LL § 33)

Commutation relations for $\hat{H},\hat{\vec{l}}^2,\hat{l}_z$ are

$$\left[\hat{H},\hat{\vec{l}}^2\right]=0,\quad \left[\hat{H},\hat{l}_z\right]=0,\quad \left[\hat{\vec{l}}^2,\hat{l}_z\right]=0.$$

Separation of variables: $\psi(\vec{r})=\psi(r,\theta,\varphi)=R(r)Y_{lm}(\theta,\varphi).$

Equation for radial part of wave equation $R(r)$ for case $U(r)=0$

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{\hbar^2 l(l+1)}{2\mu r^2}R(r)=ER(r).$$

Important relations: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)=\frac{1}{r}\frac{d^2}{dr^2}(rR)=\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}$

Replacement:

$$R(r)=\frac{\chi(r)}{\sqrt{r}};\quad R'(r)=\frac{\chi'(r)}{\sqrt{r}}-\frac{1}{2}\frac{\chi(r)}{r\sqrt{r}};\quad R''(r)=\frac{\chi''(r)}{\sqrt{r}}-\frac{\chi'(r)}{r\sqrt{r}}+\frac{3}{4}\frac{\chi(r)}{r^2\sqrt{r}}.$$

Solution: $\psi_{klm}(r)=\sqrt{\frac{2\pi k}{r}}J_{l+1/2}(kr)Y_{lm}(\theta,\varphi);\quad E_k=\frac{\hbar^2k^2}{2\mu}.$

$J_{l+1/2}(kr)$ – Bessel functions of half-integer argument, $Y_{lm}(\theta,\varphi)$ – spherical functions.

Task 3. Define energy levels of for particle movement in the field of spherically symmetrical rectangular well of finite depth U_0

$$U(r)=\begin{cases}-U_0, & r<a;\\ 0, & r>a.\end{cases}$$

First analyze the case $l=0$ (LL § 33(1)), and then, $l\neq 0$ (Flugge Z. Problems in quantum mechanics. P.1. page 168, task 63).

3. **Quiz** (~ 20 minutes). Test consists of two tasks, maximum points – **5 points.**
Hometask LL § 36(1,2)

LL – Landau L.D, Lifshits E.M, Quantum Mechnics
НКК- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Additional: Flugge Z. Problems in quantum mechanics. P.1, P.2.1974